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Prime Graph Over Semidihedral Group and Its Connectivity Indices

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Abstract

This study investigates the structural properties and topological connectivity indices of the prime graph over the semidihedral group of order 2^n, where the vertex set is defined on the entire group $V\left(\Gamma_p(SD_{2^n})\right) = SD_{2^n}$. Two distinct vertices x, y are adjacent if and only if $\gcd\left(o(x), o(y)\right)$ is a prime number. Since $o(e) = 1$ and 1 is not prime, the identity element forms an isolated vertex $\deg(e) = 0$, making $\Gamma_p(SD_{2^n})$ a disconnected graph consisting of an isolated vertex $\{e\}$ and a non-trivial connected component $\Gamma_p^*(SD_{2^n})$ of order $2^n - 1$. We partition the vertex set into three disjoint subsets: the identity element, elements of order 2, and elements of order greater than 2. Based on this partition, we characterize the degree sequence, diameter, radius, and clique number of the graph. Furthermore, exact closed-form analytical formulas are established for several fundamental degree-based and distance-based topological connectivity indices, including the First Zagreb index, Wiener index, Hyper-Wiener index, Harary index, and Forgotten index.

Keywords: Connectivity Indices, Isolated Vertex, Prime Graph, Semidihedral Group, Topological Indices

Introduction

The algebraic graph theory provides powerful framework to study structural properties of algebraic systems using graph-theoretic tools. Various graphs associated with finite groups have been introduced, including prime graphs, power graphs, coprime graphs, and enhanced power graphs [1], [2] and [3].

For a finite group G, the prime graph of G, denoted by $\Gamma_p(G)$, is defined on the vertex set $V\left(\Gamma_p(G)\right) = G$, where two distinct vertices x and y are adjacent if and only if $\gcd\left(o(x), o(y)\right)$ is a prime number [4]. Because $\gcd\left(o(e), o(x)\right) = 1$ for all $x \in G$ and 1 is not a prime number, the identity element e is not adjacent to any vertex, making e an isolated vertex in $\Gamma_p(G)$.

Topological indices are numerical invariants that quantify structural, metric, and degree-based properties of graphs. Degree-based indices such as the First Zagreb index $M_1(\Gamma)$ and the Forgotten index $F(\Gamma)$ serve as measures of degree variance [5], [6]. Distance-based indices including the Wiener index $W(\Gamma)$, Hyper-Wiener index $WW(\Gamma)$, and Harary index $H(\Gamma)$ capture distance relationships across connected components [7][8].

Recently, topological indices for various graphs associated with non-abelian groups have been investigated extensively [9], [10]. However, the structural properties and connectivity indices of the prime graph associated with the semidihedral group SD_{2^n} including the identity vertex have not yet been established.

Motivated by this, this paper establishes the structural properties of $\Gamma_p(SD_{2^n})$ with $V\left(\Gamma_p(SD_{2^n})\right) = SD_{2^n}$ and derives exact analytical expressions for its First Zagreb, Wiener, Hyper-Wiener, Harary, and Forgotten indices. To achieve this, the paper covers the following key aspects:

- **Element Order Analysis:** Characterizing the order of elements in the semidihedral group SD_{2^n} to determine vertex adjacencies.
- **Graph Structure:** Establishing the structural patterns, including the total number of vertices (order) and edges (size) of $\Gamma_p(SD_{2^n})$.
- **Index Computations:** Deriving closed-form analytical formulas for the First Zagreb, Wiener, Hyper-Wiener, Harary, and Forgotten indices.

Preliminaries

All graphs considered in this paper are simple and finite. For a graph $\Gamma, V(\Gamma)$ and $E(\Gamma)$ denote its vertex set and edge set, respectively. The degree of a vertex v, denoted by $\deg(v)$, is the number of edges incident with v.

If Γ is disconnected, its distance-based topological indices are computed on its non-trivial connected components or via finite distances. For two vertices u, v, notation $d(u, v)$ denotes the shortest path distance. If u, v lie in different components, $d(u, v) = \infty$, so $\frac{1}{d(u,v)} = 0$.

The diameter of a connected graph Γ, denoted by $\operatorname{diam}(\Gamma)$, is the maximum distance between any two vertices of Γ. The radius of Γ, denoted by $\operatorname{rad}(\Gamma)$, is the minimum eccentricity among all vertices of Γ, where the eccentricity of a vertex v is defined as the maximum distance from v to any other vertex in Γ:

$$\epsilon(v) = \max_{u \in V(\Gamma)} d\,(v, u)$$

Thus, the following inequality holds:

$$\operatorname{rad}(\Gamma) \leq \operatorname{diam}(\Gamma) \leq 2\operatorname{rad}(\Gamma)$$

A clique in Γ is a subset $C \subseteq V(\Gamma)$ such that every two distinct vertices in C are adjacent. The maximum cardinality of a clique is called the clique number of Γ, denoted by $\omega(\Gamma)$.

The order and size of Γ are denoted by $|V(\Gamma)|$ and $|E(\Gamma)|$, respectively. A graph is connected if there exists a path between every pair of vertices; otherwise, it is disconnected. A complete graph is a graph in which every pair of distinct vertices is adjacent and is denoted by K_n, where $n = |V(\Gamma)|$.

Below are the definitions and essential theorems for the connectivity indices evaluated in this study. In these definitions and subsequent discussions, the notation Γ^* denotes a connected subgraph of the graph Γ.

Definition 2.1 [5] The First Zagreb index of a graph Γ is defined as:

$$M_1(\Gamma) = \sum_{v \in V(\Gamma)} (\deg(v))^2$$

Definition 2.2 [7] The Wiener index of a graph Γ on its connected pairs is defined as:

$$W(\Gamma) = \sum_{\{u,v\} \subseteq V(\Gamma), d(u,v) < \infty} d(u, v).$$

Theorem 2.3 [7] Let Γ^* be a simple connected graph with N^* vertices and M edges. If $diam(\Gamma^*) \leq 2$, then

$$W(\Gamma^*) = 2\binom{N^*}{2} - M.$$

Definition 2.4 [8] The Hyper-Wiener index of a graph Γ on its connected pairs is defined by:

$$WW(\Gamma) = \frac{1}{2}\left(W(\Gamma) + \sum_{\{u,v\}\subseteq V(\Gamma), d(u,v)<\infty} d(u,v)^2\right).$$

Theorem 2.5 [8] Let Γ^* be a simple connected graph with N^* vertices and M edges. If $diam(\Gamma^*) \leq 2$, then

$$WW(\Gamma^*) = 3\binom{N^*}{2} - 2M.$$

Definition 2.6 [8] The Harary index of a graph Γ is defined as:

$$H(\Gamma) = \sum_{\{u,v\}\subseteq V(\Gamma), u\neq v} \frac{1}{d(u,v)}.$$

Theorem 2.7 [8] Let Γ^* be a simple connected graph with N^* vertices and M edges. If $diam(\Gamma^*) \leq 2$, then

$$H(\Gamma^*) = \frac{1}{2}\left(M + \binom{N^*}{2}\right).$$

Definition 2.8 [6] The Forgotten index (F-index) of a graph Γ is defined as:

$$F(\Gamma) = \sum_{v\in V(\Gamma)} (\deg(v))^3.$$

Result and Discussion

Algebraic Structure of Semidihedral Group

Definition 3.1 [11] The semidihedral group of order 2^n, denoted by SD_{2^n}, with $n \geq 4$, is presented by:

$$SD_{2^n} = \left\langle a, b \mid a^{2^{n-1}} = e,\ b^2 = e,\ bab^{-1} = a^{2^{n-2}-1} \right\rangle.$$

Theorem 3.2. [12] Let SD_{2^n} be the semidihedral group of order $2^n, with\ n$ ≥4. The order of each element $x \in SD_{2^n}$ is $o(e) = 1$,

$$o(a^i) = \frac{2^{n-1}}{\gcd(i, 2^{n-1})}, \quad \text{for } 1 \leq i < 2^{n-1},$$

and

$$o(a^i b) = \begin{cases} 2 & if\ i\ even \\ 4 & if\ i\ odd \end{cases}$$

From Theorem 3.2, the elements of order 2 in SD_{2^n} are $a^{2^{n-2}}$ and $a^i b$ for even i, giving a total of $2^{n-2} + 1$ elements of order 2. Specifically, within the cyclic subgroup $\langle a\rangle$, the unique element of order 2 is $a^{2^{n-2}}$, as solving

$$gc\,\mathrm{d}(i, 2^{n-1}) = 2^{n-2} \text{ yields } i = 2^{n-2} \text{ for } 1 \leq i < 2^{n-1}.$$

Meanwhile, for elements of the form $a^i b$, the condition $o(a^i b) = 2$ holds if and only if i is even. Since $i \in \{0,1,\ldots,2^{n-1} - 1\}$, exactly half of these 2^{n-1} indices are even, yielding 2^{n-2} such elements. Summing these two disjoint sets gives

$$|V(\Gamma)| = 2^{n-2} + 1.$$

Consequently, the order of this graph Γ directly governs its structural invariants, including the clique number $\omega(\Gamma)$, diameter $\mathrm{diam}(\Gamma)$, and radius $\mathrm{rad}(\Gamma)$

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Structure of Prime Graph Over Semidihedral Group

Definition 3.3 Let G be a finite group. The prime graph of G, denoted by $\Gamma_p(G)$, is the simple graph with vertex set $V\left(\Gamma_p(G)\right) = G$, where distinct vertices x, y are adjacent if and only if $\gcd\big(o(x), o(y)\big)$ is a prime number.

We partition $V\left(\Gamma_p(SD_{2^n})\right) = SD_{2^n}$ into three mutually disjoint subsets:

$$E = \{e\},$$
$$U = \{x \in SD_{2^n} \mid o(x) = 2\},$$
$$W = \{x \in SD_{2^n} \mid o(x) \geq 4\}.$$

The cardinalities of these subsets are:

$$|E| = 1, \qquad |U| = 2^{n-2} + 1, \qquad |W| = 3(2^{n-2}) - 2.$$

Note that $|E| + |U| + |W| = 1 + (2^{n-2} + 1) + (3(2^{n-2}) - 2) = 2^n$.

Example 3.4 Consider SD_{16} $n = 4$. We have $\left|V\left(\Gamma_p(SD_{16})\right)\right| = 16$. Here

$$E = \{e\},$$
$$U = \{a^4, b, a^2b, a^4b, a^6b\}, |U| = 5$$
$$W = \{a, a^2, a^3, a^5, a^6, a^7, ab, a^3b, a^5b, a^7b\}$$

with $|W| = 10$. The structure of $\Gamma_p(SD_{16})$ is shown in Figure 1.

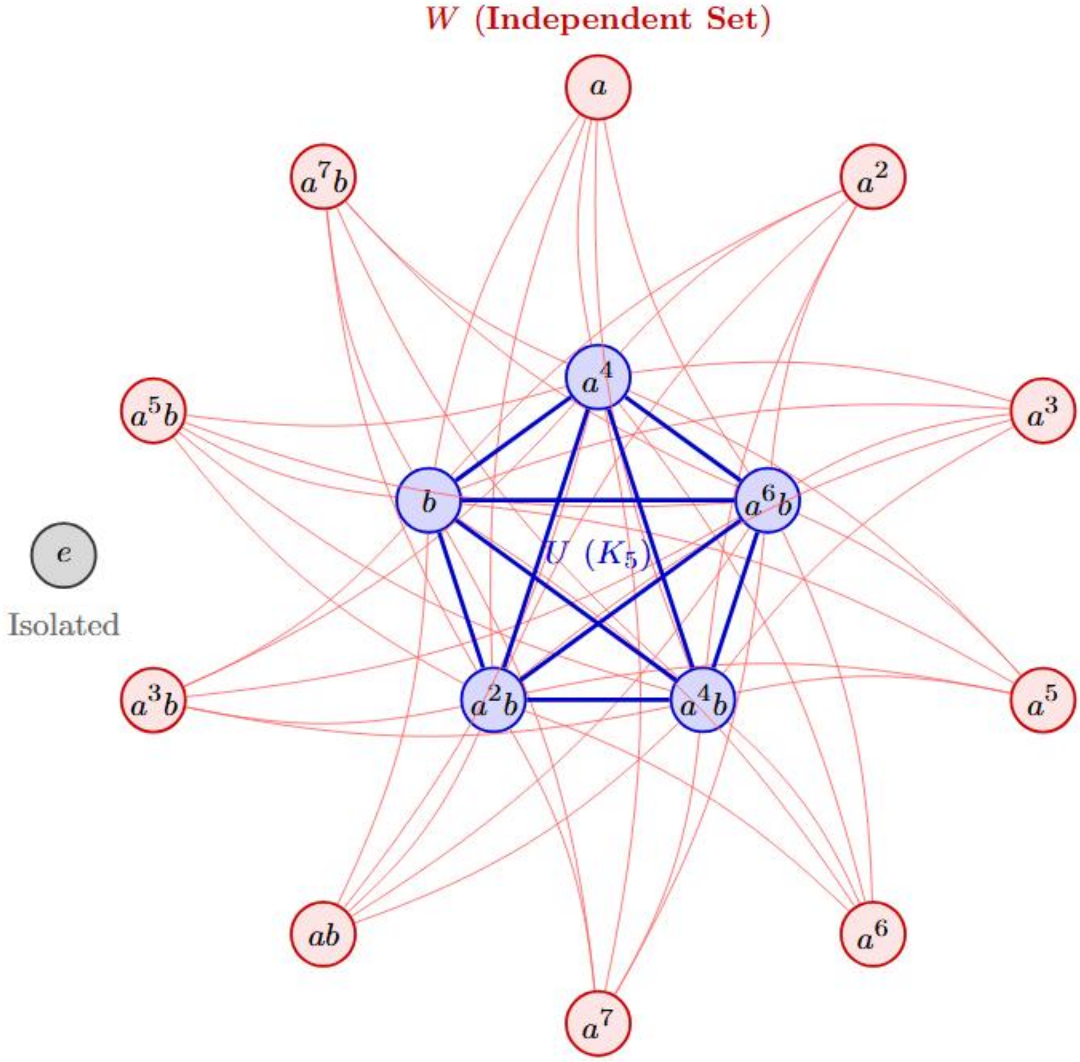

Figure 1. Prime Graph of SD_{16}.

Theorem 3.5 Let $\Gamma_p(SD_{2^n})$ be the prime graph of SD_{2^n} $(n \geq 4)$. Then the degree of each vertex is given by:

$$\deg(v) = \begin{cases} 0 & if\ v = e \\ 2^n - 2 & if\ v \in U \\ 2^{n-2} + 1 & if\ v \in W \end{cases}$$

Proof. For $v = e$, $o(e) = 1$. For any $x \in SD_{2^n} \setminus \{e\}$, $\gcd\big(o(e), o(x)\big) = 1$, which is not prime. Thus, $\deg(e) = 0$.

Based on Theorem 3.2, for each $u \in U, o(u) = 2$. Since for any other non-identity vertex $x \in (U \cup W) \setminus \{u\}$, $o(x) = 2^k$ $(k \geq 1)$. Thus, $\gcd\big(o(u), o(x)\big) = 2$, which is prime. So, each vertex $u \in U$ adjacent to all $2^n - 2$ non-identity elements. Therefore $\deg(u) = 2^n - 2$.

Based on Theorem 3.2, for each, for $w \in W, o(w) \geq 4$. If $v \in U$, $\gcd\big(o(w), o(v)\big) = 2$ (prime), so w is adjacent to all elements in U. If $v \in$, $\gcd\big(o(w), o(v)\big)$ ≥4 (not prime). If $v = e$, $\gcd\big(o(w), o(e)\big) = 1$ (not prime). Therefore, w is adjacent only to vertices in U, giving

$$deg(W) = |U| = 2^{n-2} + 1.$$

This completes the proof. ■

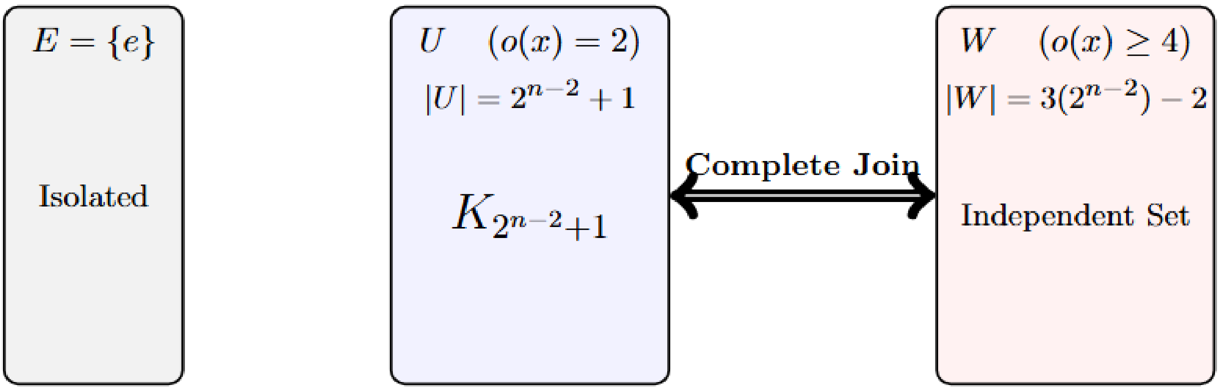

Figure 2. Decomposition of $\Gamma_p(SD_{2^n})$ into isolated vertex e and connected component Γ_p^*

Corollary 3.6. Let $\Gamma_p(SD_{2^n})$ be the prime graph of SD_{2^n} $(n \geq 4)$. Then:

1. $\Gamma_p(SD_{2^n})$ is disconnected and consists of two components: $\{e\}$ and $\Gamma_p^*(SD_{2^n})$
2. $rad\left(\Gamma_p^*(SD_{2^n})\right) = 1$
3. $diam\left(\Gamma_p^*(SD_{2^n})\right) = 2,$
4. $\omega\left(\Gamma_p(SD_{2^n})\right) = 2^{n-2} + 2.$

Proof. Since $o(e) = 1$ and 1 is not a prime number, $\gcd\big(o(e), o(x)\big) = 1$ for all $x \in SD_{2^n} \setminus \{e\}$. Thus, $\deg(e) = 0$, meaning $\{e\}$ forms an isolated vertex. Meanwhile, since $|U| = 2^{n-2} + 1 \geq 5$ for $n \geq 4$, every vertex in U is adjacent to all other non-identity elements in $U \cup W$. Hence, the remaining $2^n - 1$ vertices form a single non-trivial connected component, denoted by $\Gamma_p^*(SD_{2^n})$.
The eccentricity of a vertex $v \in V\big(\Gamma_p^*\big)$ is defined as

$$\varepsilon(v) = \max\{\, d(u, v) \mid u \in V\big(\Gamma_p^*\big)\}$$

For any $u \in U, u$ is adjacent to every other vertex in $V\big(\Gamma_p^*\big)$, so $d(u, v) = 1$ for all $v \neq u$. Thus, $\varepsilon(u) = 1$ for all $u \in U$. The radius of Γ_p^* is the minimum eccentricity:

$$rad\big(\Gamma_p^*\big) = \min_{v \in V(\Gamma_p^*)} \varepsilon\,(v) = \varepsilon(u) = 1.$$

For any two distinct vertices $w_1, w_2 \in W$, $\gcd\big(o(w_1), o(w_2)\big) \geq 4$, which is not prime, so w_1 and w_2 are not adjacent. However, both w_1 and w_2 are adjacent to any vertex $u \in U$. Thus, the shortest path between them is $w_1 \sim u \sim w_2$, giving $d(w_1, w_2) = 2$. Consequently, $\varepsilon(w) = 2$ for all $w \in W$. The diameter of Γ_p^* is the maximum eccentricity:

$$diam(\Gamma_p^*) = \max_{v \in V(\Gamma_p^*)} \varepsilon(v) = \varepsilon(w) = 2.$$

A clique is a subset of vertices inducing a complete subgraph. The subset U induces a complete graph $K_{|U|}$. Since W is an independent set, no two vertices in W are adjacent to each other. Thus, a maximum clique can include all $|U|$ vertices from U, but at most one vertex from W. Adding any single vertex $w \in W$ to U forms a complete graph of order $|U| + 1$ because w is adjacent to every vertex in U. Therefore, the clique number is:

$$\omega\left(\Gamma_p(SD_{2^n})\right) = |U| + 1 = (2^{n-2} + 1) + 1 = 2^{n-2} + 2.$$

This completes the proof. ■

Connectivity Indices of Prime Graph over Semidihedral Group

This section discusses the connectivity indices of the prime graph of the semidihedral group. The connectivity indices evaluated include the First Zagreb, Wiener, Hyper-Wiener, Harary, and Forgotten indices. The computation of the Wiener, Hyper-Wiener, and Harary indices will utilize theorems concerning the number of vertices and edges in the constructed graph.

Let $x = 2^{n-2}$. The cardinalities of the vertex partitions are $|E| = 1, |U| = x + 1$, and $|W| = 3x - 2$. The total number of vertices in $\Gamma_p(SD_{2^n})$ is $N = 4x$, while the order of the main connected component $\Gamma_p^*(SD_{2^n})$ is $N^* = 4x - 1$.

By the Handshaking Lemma, the total number of edges M is computed as:

$$\begin{aligned} M = \frac{1}{2} \sum_{v \in V(\Gamma_p)} \deg(v) &= \frac{1}{2}\big(\deg(e) + |U|(4x-2) + |W|(x+1)\big) \\ &= \frac{1}{2}\big(0 + (x+1)(4x-2) + (3x-2)(x+1)\big) = \frac{1}{2}(x+1)\big((4x-2) + (3x-2)\big) \\ &= \frac{1}{2}(x+1)(7x-4) = \frac{7x^2 + 3x - 4}{2} \end{aligned}$$

Theorem 3.7 (First Zagreb Index) Let $\Gamma_p(SD_{2^n})$ be the prime graph of SD_{2^n} $(n \geq 4)$. Then:

$$M_1\left(\Gamma_p(SD_{2^n})\right) = 19(2^{3n-6}) + 2^{2n-2} - 13(2^{n-2}) + 2$$

Proof. By definition, the First Zagreb index is the sum of squared degrees over all vertices:

$$M_1(\Gamma_p) = (\deg(e))^2 + \sum_{u \in U} (\deg(u))^2 + \sum_{w \in W} (\deg(w))^2$$

Substituting the degree values $\deg(e) = 0$, $\deg(u) = 4x - 2$, and $\deg(w) = x + 1$:

$$\begin{aligned} M_1(\Gamma_p) &= 0^2 + |U|(4x-2)^2 + |W|(x+1)^2 = (x+1)(16x^2 - 16x + 4) + (3x-2)(x^2 + 2x + 1) \\ &= (16x^3 - 16x^2 + 4x + 16x^2 - 16x + 4) + (3x^3 + 6x^2 + 3x - 2x^2 - 4x - 2) \\ &= (16x^3 - 12x + 4) + (3x^3 + 4x^2 - x - 2) \\ &= 19x^3 + 4x^2 - 13x + 2 \end{aligned}$$

Substituting $x = 2^{n-2}$ back into the polynomial expression, we have $x^3 = 2^{3n-6}$, $4x^2 = 4(2^{2n-4}) = 2^{2n-2}$, and $x = 2^{n-2}$. This yields:

$$M_1\left(\Gamma_p(SD_{2^n})\right) = 19(2^{3n-6}) + 2^{2n-2} - 13(2^{n-2}) + 2$$

This completes the proof.■

Theorem 3.8 (Wiener Index) Let $\Gamma_p(SD_{2^n})$ be the prime graph of SD_{2^n} $(n \geq 4)$. Then:

$$W\left(\Gamma_p(SD_{2^n})\right) = 25(2^{2n-5}) - 27(2^{n-3}) + 4$$

Proof. Since e is an isolated vertex, $d(e, v) =$ for all $v \neq e$, so distance-based indices are evaluated over connected pairs in the main component Γ_p^*. In Γ_p^*, the total number of unordered vertex pairs is

$$\binom{N^*}{2} = \binom{4x-1}{2} = 8x^2 - 6x + 1.$$

Since $\mathrm{diam}\left(\Gamma_p^*\right) = 2$, then using the Theorem 2.3 we got the Weiner index as below:

$$W(\Gamma_p) = 1 \cdot M + 2\left(\binom{N^*}{2} - M\right) = 2\binom{N^*}{2} - M$$
$$= 2(8x^2 - 6x + 1) - \left(\frac{7x^2 + 3x - 4}{2}\right)$$
$$= 16x^2 - 12x + 2 - \frac{7}{2}x^2 - \frac{3}{2}x + 2$$
$$= \frac{25}{2}x^2 - \frac{27}{2}x + 4$$

Substituting $x = 2^{n-2}$ gives $x^2 = 2^{2n-4}$. Simplifying the fractions:

$$W\left(\Gamma_p(SD_{2^n})\right) = \frac{25}{2}(2^{2n-4}) - \frac{27}{2}(2^{n-2}) + 4 = 25(2^{2n-5}) - 27(2^{n-3}) + 4$$

This completes the proof.■

Theorem 3.9 (Hyper-Wiener Index) Let $\Gamma_p(SD_{2^n})$ be the prime graph of $SD\_\{2\hat{}n\}$ $(n \geq 4)$. Then:

$$WW\left(\Gamma_p(SD_{2^n})\right) = 17(2^{2n-4}) - 21(2^{n-2}) + 7$$

Proof. Since $\mathrm{diam}\left(\Gamma_p^*\right) = 2$, then using the Theorem 2.5 we got the Hyper- Weiner index as below:

$$WW(\Gamma_p) = 3\binom{N^*}{2} - 2M$$
$$= 3(8x^2 - 6x + 1) - 2\left(\frac{7x^2 + 3x - 4}{2}\right)$$
$$= (24x^2 - 18x + 3) - (7x^2 + 3x - 4)$$
$$= 17x^2 - 21x + 7$$

Substituting $x = 2^{n-2}$ yields the analytical formula directly:

$$WW\left(\Gamma_p(SD_{2^n})\right) = 17(2^{2n-4}) - 21(2^{n-2}) + 7$$

This completes the theorem. ■

Theorem 3.10 (Harary Index) Let $\Gamma_p(SD_{2^n})$ be the prime graph of SD_{2^n} $(n \geq 4)$. Then:

$$H\left(\Gamma_p(SD_{2^n})\right) = 23(2^{2n-6}) - 9(2^{n-4}) - \frac{1}{2}$$

Proof. Since $\mathrm{diam}\left(\Gamma_p^*\right) = 2$, then using the Theorem 2.7 we got the Harary index as below:

$$H(\Gamma_p) = \frac{1}{2}\left(M + \binom{N^*}{2}\right)$$
$$= \frac{1}{2}\left(\frac{7x^2 + 3x - 4}{2} + (8x^2 - 6x + 1)\right)$$

$$= \frac{1}{2}\left(\frac{7x^2 + 3x - 4 + 16x^2 - 12x + 2}{2}\right)$$
$$= \frac{1}{2}\left(\frac{23x^2 - 9x - 2}{2}\right) = \frac{23}{4}x^2 - \frac{9}{4}x - \frac{1}{2}$$

Substituting $x = 2^{n-2}$ gives $x^2 = 2^{2n-4}$. Dividing the coefficients by 4:

$$H\left(\Gamma_p(SD_{2^n})\right) = \frac{23}{4}(2^{2n-4}) - \frac{9}{4}(2^{n-2}) - \frac{1}{2} = 23(2^{2n-6}) - 9(2^{n-4}) - \frac{1}{2}$$

This completes the proof. ■

Theorem 3.11 (Forgotten Index) Let $\Gamma_p(SD_{2^n})$ be the prime graph of SD_{2^n} ($n \geq 4$). Then:

$$F\left(\Gamma_p(SD_{2^n})\right) = 67(2^{4n-8}) - 25(2^{3n-6}) - 45(2^{2n-4}) + 37(2^{n-2}) - 10$$

Proof. According to definition, the Forgotten index is the sum of cubed degrees over all vertices:

$$F(\Gamma_p) = (\deg(e))^3 + \sum_{u \in U} (\deg(u))^3 + \sum_{w \in W} (\deg(w))^3$$

Substituting the vertex degrees and expanding each binomial term:

$$F(\Gamma_p) = 0^3 + |U|(4x - 2)^3 + |W|(x + 1)^3$$
$$= (x + 1)(64x^3 - 96x^2 + 48x - 8) + (3x - 2)(x^3 + 3x^2 + 3x + 1)$$

Expanding both polynomial products separately:

$$(x + 1)(64x^3 - 96x^2 + 48x - 8) = 64x^4 - 32x^3 - 48x^2 + 40x - 8$$
$$(3x - 2)(x^3 + 3x^2 + 3x + 1) = 3x^4 + 7x^3 + 3x^2 - 3x - 2$$

Adding these two expressions term-by-term yields:

$$F(\Gamma_p) = 67x^4 - 25x^3 - 45x^2 + 37x - 10$$

Substituting $x = 2^{n-2}$ gives $x^4 = 2^{4n-8}$, $x^3 = 2^{3n-6}$, $x^2 = 2^{2n-4}$, and $x = 2^{n-2}$, completing the proof:

$$F\left(\Gamma_p(SD_{2^n})\right) = 67(2^{4n-8}) - 25(2^{3n-6}) - 45(2^{2n-4}) + 37(2^{n-2}) - 10. ■$$

Conclusion

In this paper, we have completely characterized the structural properties and calculated fundamental topological connectivity indices of the prime graph $\Gamma_p(SD_{2^n})$ over the semidihedral group including the identity vertex $\left(V\left(\Gamma_p(SD_{2^n})\right) = SD_{2^n}\right)$. Because $o(e) = 1$ and 1 is not prime, the identity element acts as an isolated vertex ($\deg(e) = 0$), making the graph disconnected with one isolated vertex $\{e\}$ and one non-trivial connected component $\Gamma_p^*(SD_{2^n})$ of order $2^n - 1$. The component Γ_p^* exhibits a split-like structure where a central clique of elements of order $2(|U| = 2^{n-2} + 1)$ is completely joined to an independent set of elements of order $n \geq 4(|W| = 3(2\text{^}\{n-2\}) - 2)$. Closed-form formulas for the First Zagreb index, Wiener index, Hyper-Wiener index, Harary index, and Forgotten index were successfully established.

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