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Probabilistic Inventory Model Using All Unit Discount Factor and Inflation

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Abstrak

Inflasi menyebabkan harga per-satuan barang naik di masa depan. Untuk mengimbangi inflasi, perusahaan dapat memanfaatkan *discount* yang diberikan oleh *supplier*. Tetapi pemesanan barang harus disesuaikan dengan permintaan untuk menghindari peningkatan biaya penyimpanan. Penelitian ini menjelaskan mengenai model inventory probabilistik dengan permintaan pada waktu tunggu berdistribusi Gamma dan mempertimbangkan adanya faktor discount dan inflasi. Kondisi *backorder* diperbolehkan dalam pembentukan model ini. Penelitian ini menghasilkan bentuk model inventori probalilistik dan algoritma untuk mendapatkan jumlah pemesaann optimal dan waktu pemesanan optimal untuk barang-barang yang *all unit discount* dan mengalami inflasi dengan permintaan berdistibusi Gamma.

Kata Kunci: Model *Inventory* Probabilistik Gamma; Model *Inventory Discount* dan Inflasi; Model *Inventory Backorder*.

Abstract

Inflation causes the price per unit of goods to rise in the future. To offset inflation, companies can take advantage of discounts provided by suppliers. But ordering goods must be adjusted according to demand to avoid storage costs. This study describes a probabilistic supply model with demand at a time with a Gamma distribution and waiting for the discount and inflation factors. Backorder conditions in this model is assumed. The research produces a probabilistic inventory model and an algorithm to obtain the optimal number of orders and optimal ordering times for items that are all discounted and inflated with Gamma distribution demand.

Keywords: Gamma Inventory Probabilistic Model; Inventory Model with Discount and Inflation; Model with Backorder

Introduction

Inventory is one of the factors that can affect the profits earned by a company. Good inventory management can minimize the company's operational costs and avoid waste or inventory shortages. One of the factors that companies must pay attention to in managing inventory is to consider the inflation factor. Inflation can cause an increase in the price of the unit of goods so that the total cost of inventory will also increase. To minimize the total cost of inventory due to an increase in the price of goods, companies can take advantage of discount offers provided by suppliers to the company.

Paweł Ślaski [1] has discussed mathematical modeling for optimal purchasing by considering the existence of inflation caused by stagnation of supply and increase in employee salaries. Kal Namit and Jim Chen [2] have previously discussed the inventory model with changing demand with a Gamma distribution. Then the same study was carried out by John E Tywort and Ram Geneshan [3]. However, both studies only discuss demand that is not constant. In addition, Onayumi et al. [4] in their research discusses the development of the EOQ model by running out of stock considering the discount and inflation factors, but in this study the demand is considered constant. Another author Valliathal Uthayakuma [5] discusses the Weibull distribution model of supply with respect to the inflation factor.

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In this paper we are interested in conducting research in the development of an inventory model with non-constant demand conditions with regard to the discount and inflation.

Research Method

The research method used in this research is a literature study with the following steps:

1. Consider we have probabilistic inventory model probabilistik as follows:
   a. Purchase costs $= DP$. (1)
   b. Order fee $= \frac{AD}{Q}$. (2)
   c. Storage costs $= H\left[\frac{Q}{2} + R - E(X)\right]$. (3)
   d. Shortage costs $= \frac{WD}{Q}\left[\int_R^\infty (x-R)f(x)dx\right]$. (4)
   e. Total inventory cost =
   $$TAC\ (Q,R) = DP + \frac{AD}{Q} + H\left[\frac{Q}{2} + R - E(X)\right] + \frac{WD}{Q}\left[\int_{-\infty}^\infty (x-R)f(x)dx\right]. \quad (5)$$
2. Let an inventory model with a discount on all units. The discount equation as follows:
   $$TAC\ (0,L) = \begin{cases} TAC(y_m) & if \quad y_m \le q_1 \\ TAC\ (q_1) & if \quad q_1 \le y_m \le q_2. \\ TAC\ (q_2) & if \quad q_2 \le y_m \end{cases} \quad (6)$$
3. Consider an inventory model with an inflation factor.
   The inventory model with an inflation factor considers the rate of increase in the overall cost price per unit of time, then:
   $$C(T) = C_0 e^{kT}. \quad (7)$$
4. Let Gamma distribution.
   The probability function of the Gamma distribution is defined as:
   $$f(x) = \frac{1}{\beta^\alpha \Gamma(\alpha)} x^{\alpha-1} e^{-\frac{x}{\beta}} \quad (8)$$
   with parameters $\alpha > 0, \beta > 0$, and $\Gamma(\alpha) = (\alpha - 1)!$
5. Based on (1), (2), (3) and (4), a probabilistic inventory model is formed by considering the all unit discount factor and inflation.
6. Based on (5) will be found the optimal order quantity Q* and the reorder point R^*.
7. Conclusion.

Results and Discussion

Furthermore, a probabilistic inventory model will be formed with the factor of all unit discount and inflation, with the demand for goods at the waiting time distributed in Gamma.

The assumptions required in the formation of this inventory model are:

1. The level of demand for goods during waiting time is Gamma distributed.
2. There is no limited storage warehouse.
3. Lead-time should be constant.
4. Shortage of goods occurs when the demand for goods is greater than the number of goods in the warehouse at the lead-time.
5. Requests that are not satisfied by the lead-time will be fulfilled in the next period or backorder is allowed.
6. The inflation rate should be constant.

7. The type of discount given is the all unit discount.
8. The number of orders placed is always the same for each order submitted.
9. The storage cost depends on the average of the items stored.
10. The length of each booking period is the same for each next booking period.

The notations used in the formation of this inventory model are:

D	: The average amount of demand for goods per year.
A	: Order fee for each time an order is submitted.
H	: Storage cost per unit of goods per year.
W	: Shortage cost per unit of goods.
P	: The purchase price of goods per unit.
Q	: The optimal order quantity.
R	: Reorder point.
TAC	: Total inventory cost.
$f(x)$	: The opportunity density function of demand during the lead time.
C_1	: Purchase costs.
C_2	: Order fee.
C_3	: Storage costs.
C_4	: Shortage costs.
$C(T)$	: The rate of incremental costs over the total cost per unit of time.
L	: One planning period (1 year).
T	: One period.
l	: The number of orders made in one year.
k	: Inflation rate.
t	: Lead-time.
ss	: Safety Level.
μ	: Expected lead-time demand.

The probabilistic inventory model that is affected by the inflation rate can be formulated as follows:

1. Order fee (C_2)

Ordering costs are costs incurred when the company places an order for goods to be stored in inventory. Order fees can be in the form of telp fees, correspondence fees and so on. The amount of the ordering fee for one year is:

$$\begin{aligned} C_2 &= \text{one} - \text{time order} \times \text{ordering frequency} \\ &= \frac{AD}{Q}. \end{aligned}$$

Then, we know the inflation equation in the period (T) is presented as follows [6]:

$$C_0(T) = C_0 e^{kT}.$$

T is one order period. This model plans an order planning (L) for one year, dilakukan selama satu tahun, then $L = 1$ year. Based on the inflation equation above, the cost of ordering which is influenced by inflation in period (T) can be denoted as follows:

$$C_2(T) = Ae^{kT}. \tag{9}$$

If the order is for 1 year, then:

$$L = lT. \tag{10}$$

The value $l = \frac{D}{Q}$ is the number of times the order is made per one planning period L (one year). Then the cost of ordering during one planning period $(0, L)$ is:

$$C_2(0, L) = A(0T) + A(T) + A(2T) + \cdots + A((l-1)T). \tag{11}$$

Moreover, based on Equation (9) and (11) then it is obtained the ordering cost for one year is:

$$C_2(0,L) = \left[A + Ae^{kT} + Ae^{2kT} + \cdots + Ae^{(l-1)kT}\right] = \sum_{n=0}^{l-1} Ae^{kTn}. \tag{12}$$

Equation (12) is a geometric series, to simplify the equation, we multiply the both side in Equation (12) with e^{kT}, yields:

$$e^{kT}C_2(0,L) = \left[Ae^{kT} + Ae^{2kT} + \cdots + Ae^{(l-1)kT} + Ae^{lkT}\right]. \tag{13}$$

Then, based on Equation (10), Equation (13) can be rewritten as follows:

$$e^{kT}C_2(0,L) = \left[Ae^{kT} + Ae^{2kT} + \cdots + Ae^{(l-1)kT} + Ae^{kL}\right]. \tag{14}$$

The cost of ordering during one planning period which is affected by inflation can be obtained from the difference in Equation (12) and Equation (14), which is as follows:

$$C_2(0,L) - e^{kT}C_2(0,L) = A - Ae^{kL}$$
$$C_2(0,L)(1 - e^{kT}) = A(1 - e^{kL})$$
$$C_2(0,L) = \frac{A(1 - e^{kL})}{(1 - e^{kT})}$$
$$= A\left(\frac{e^{kL}-1}{e^{kT}-1}\right). \tag{15}$$

2. Purchase costs (C_1)

Purchasing costs are costs incurred by the company to purchase goods that will be stored in inventory. With the same steps for ordering costs, the purchase costs that are affected by inflation for one year are obtained as follows:

$$C_1(0,L) = QP\left(\frac{e^{kL}-1}{e^{kT}-1}\right). \tag{16}$$

3. Storage costs (C_3)

Storage costs are costs incurred by the company due to inventory storage activities. Storage costs can be in the form of warehouse rental fees, maintenance costs for goods, insurance costs, tax fees and so on. Similar steps for ordering costs, the storage costs that are affected by inflation for one year are obtained as follows:

$$C_3(0,L) = H\left[\frac{Q}{2} + R - \alpha\beta\right]\left(\frac{e^{kL}-1}{e^{kT}-1}\right). \tag{17}$$

4. Shortage costs

Shortage costs are costs that must be incurred by the company due to unfulfilled customer requests. Similar steps for ordering costs, the cost of the shortage which is affected by inflation for one year is obtained. Shortage costs can be resolved in two ways from the parameter α. If $\alpha = 1$ then the shortage costs which be affected by inflation during the planning period $(0, L)$ can be presented as follows:

$$C_{4(\alpha=1)}(0,L) = W\beta e^{-\frac{R}{\beta}}\left(\frac{e^{kL}-1}{e^{kT}-1}\right). \tag{18}$$

If $\alpha > 1$, then the shortage costs that influenced by inflation during the planning period $(0, L)$ can be granted as follows:

$$C_{4(\alpha>1)}(0,L) = W\left(\sigma_L G_u(ss)\right)\left(\frac{e^{kL}-1}{e^{kT}-1}\right). \tag{19}$$

5. Total inventory costs

After obtaining the formulation of inventory costs, then the total inventory cost will be searched for the probabilistic inventory model which is influenced by inflation. So that the total inventory cost of the probabilistic inventory model with the inflation factor is obtained. The demand at the lead-time for the Gamma distribution when $\alpha = 1$ is as follows:

$$TAC_{(\alpha=1)} = C_1(0,L) + C_2(0,L) + C_3(0,L) + C_{4(\alpha=1)}(0,L)$$

$$= \left(QP + A + H\left[\frac{Q}{2} + R - \alpha\beta\right] + W\beta e^{-\frac{R}{\beta}}\right)\left(\frac{e^{kL}-1}{e^{kT}-1}\right). \quad (20)$$

Next, to obtain the minimum total inventory cost, then we have to find the optimal value of $Q^*$and R^* , which is by derivate Equation (20) partially with respect to $\frac{\partial TAC}{\partial Q}$ = 0 and $\frac{\partial TAC}{\partial R}$ = 0. Obtained:

$$R^* = -\beta \ln \frac{H}{W} \quad (21)$$

$$Q^* = T \times D \quad (22)$$

Furthermore, a probabilistic inventory model will be formed which is influenced by the inflation actor, the demand for goods at the leading-time with a Gamma distribution with parameter $\alpha > 1$, as follows:

$$TAC_{(\alpha>1)} = (C_1(0,L) + C_2(0,L) + C_3(0,L) + C_{4(\alpha>1)}(0,L))$$

$$= \left(QP + A + H\left[\frac{Q}{2} + R - \alpha\beta\right] + W(\sigma_L G_u(ss))\right)\left(\frac{e^{kL}-1}{e^{kT}-1}\right) \quad (23)$$

Moreover, to get the minimum total cost of inventory, the optimal Q^* will be founded by deriving Equation (20) partially with respect to $\frac{\partial TAC}{\partial Q}$ = 0 so that it is obtained:

$$Q^* = T \times D \quad (24)$$

The algorithm for obtaining the optimal number of orders for goods and reordering points for a probabilistic inventory model with an all-unit discount and inflation factor is presented as follows:

1. Determine the known value of parameter α.
2. Calculating Q_0 using $Q_0 = \sqrt{\frac{2DA}{H}}$
3. Calculating R^* using Equation (18) for parameter $\alpha = 1$ and Equation (19) for parameter $\alpha > 1$.
4. Calculating Q^* using $Q^* = T^* \times D$.
5. Calculate the total cost of inventory using the equation that matches the known value of the α parameter.
6. Compare the value of the total cost of inventory with the total value of inventory obtained if the value of Q is used in the first discount equation, then Q is valid (showing the minimum total cost of inventory).
7. If Q is not valid, then
   (i) Use Q value at the second equation.
   (ii) Use Q value at third equation.
8. Choose the Q order quantity that gives the minimum total inventory cost.

Numerical Simulations

An illustration: A company that supplies car spare parts requires an average of 500 units of goods per year. The company must pay an order fee of Rp.300,000 / order; and the cost of storing goods is Rp.30,000 / unit, while the cost that must be incurred by the company if there is a shortage of goods is Rp.200,000 / unit. The inflation rate is known to increase by 25% in each order and demand order at the Gamma distribution waiting time. The supplier gives a discount to the company with the following offer;

Table 1. Discount to Company

Unit	Discount	Price/Unit
$80 > Q$	Normal	Rp.350.000;
$80 \leq Q < 100$	5%	Rp.332.500;
$100 \leq Q < 120$	10%	Rp.315.000;
$120 \leq Q$	15%	Rp. 297.500;

A. For parameter $\alpha = 1$ and $\beta = 1$.

Using the previous algorithm, obtained $Q = 78$, $R^* = 2$, $Q^* = 77{,}5$ and inventory cost component:

Table 2. Inventory Cost Component

	Costs without the effect of inflation / discount	Costs are influenced by inflation
Purchase costs in a year $C_1(0{,}1 \text{ year})$	191.100.000	217.581.000
Order fee in a year $C_2(0{,}1 \text{ year})$	2.100.000	2.391.000
Storage costs in a year $C_3(0{,}1 \text{ year})$	8.820.000	100.422
Shortage costs in a year $C_4(0{,}1 \text{ year})$	25.641,84	29.195,066
Total inventory costs for the year $TAC(0{,}1 \text{ year})$	202.045.641,8	230.043.395,1

To minimize the total cost of inventory that increases due to the influence of inflation, companies can take advantage of discount offers provided by suppliers, as follows:

Table 3. *TAC* influenced by inflation and discount

Q (Unit)	l (Lots of orders)	T(Length of period / year)	TAC
80	7	0,14	224.411.824,5
100	5	0,2	182.805.580
120	5	0,2	206.031.580,8

It can be seen that the increase in inventory costs caused by the influence of inflation was quite large, namely from Rp.202.045.641,8; to Rp.230.043.395,1; annually. Then the company can minimize the cost of supplies if the company buys 100 units of goods / order at a 10% discount when the remaining inventory is 2 units and the order period is 0.2 years or 2.4 months or 72 days. The inventory cost is thus Rp. 182.805.580,0; annually.

B. For parameter $\alpha > 1$, such as $\alpha = 15$ and $\beta = 1$, standard deviation of product demand (σ) is 10 unit/week and the company want at most a 2% chance of a shortage of goods during the lead-time.

Using the previous algorithm, obtained $ss = 2{,}06$, $R = 40{,}6$, $G_u(ss) = 17{,}94$, $Q^* = 77{,}5$ respectively and $TAC = 410.153.076.9$ (without being affected by inflation), $TAC = 531.678.700$ (influenced by inflation).

To minimize the total cost of inventory that increases due to the influence of inflation, companies can take advantage of discount offers provided by suppliers, as follows:

Table 4. *TAC* influenced by inflation

Q (Unit)	l (Lots of orders)	T(Length of period / year)	TAC
80	7	0,14	526.099.700
100	5	0,2	392.132.3000
120	5	0,2	415.385.300

It can be seen that the increase in inventory costs caused by the influence of inflation was quite large, namely from Rp. 410.153.076,9; to Rp. 531.678.700; annually. Then the company can minimize the cost of supplies if the company buys 100 units of goods / orders with a 10% discount when the remaining inventory is 41 units and the order period is 0,2 years or 2,4 months or 72 days. The inventory cost is thus Rp. 392.132.300; annually.

Conclussion

The increase in inventory costs can be minimized by taking advantage of discount offers by suppliers. Generally, the discount amount is adjusted to the amount of goods purchased by the company in the form of a scheme. In the model simulation for $\alpha \geq 1$ and $\beta = 1$ it is shown that to determine the minimum cost of inventory, you must pay attention to the amount of discount, the number of units purchased, and the time period for ordering.

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